Synthesis of Planar 4 Bar Mechanisms

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Contents

Chapter 1

Problem Statement

The Project Aims to implement the 5 pose Burmester problem for a Four bar linkage using algorithm proposed by K. Brunnthaler, M. Pfurner, and M. Husty. in their research paper "Synthesis of planar four-bar mechanisms".Their algorithm does not ensure that all the poses lie in the same branch i.e. branch defects may be present.

Chapter 2

Theory

2.1 Planar kinematic mapping

First, we map the Fixed link frame to moving coupler frame using kinematic mapping.

$$\mathbf{p_0} = \mathbf{A} \cdot \mathbf{p} \qquad \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ a & \cos\phi & -\sin\phi \\ b & \sin\phi & \cos\phi \end{pmatrix}$$

$$\left(2\cos\frac{\phi}{2} : 2\sin\frac{\phi}{2} : a\sin\frac{\phi}{2} - b\cos\frac{\phi}{2} : a\cos\frac{\phi}{2} + b\sin\frac{\phi}{2}\right) = (X_0 : X_1 : X_2 : X_3).$$

$$\begin{pmatrix} Z \\ X \\ Y \end{pmatrix} = \begin{pmatrix} X_0^2 + X_1^2 & 0 & 0 \\ 2(X_1X_2 + X_0X_3) & X_0^2 - X_1^2 & -2X_0X_1 \\ 2(X_1X_3 - X_0X_2) & 2X_0X_1 & X_0^2 - X_1^2 \end{pmatrix} \begin{pmatrix} z \\ x \\ y \end{pmatrix}$$

2.2 Mechanism analysis

Then, we calculate the constraints curve equation for a circle and also generalise it for its degenerate case i.e. a line.

$$C_0(X^2 + Y^2) - 2C_1XZ - 2C_2YZ + (C_1^2 + C_2^2 - R^2)Z^2 = 0$$

$$(R^2 - C_1^2 - C_2^2 - C_0(x^2 + y^2) + 2C_1x + 2C_2y)X_0^2 + (R^2 - C_1^2 - C_2^2 - C_0(x^2 + y^2) - 2C_1x - 2C_2y)X_1^2$$

$$+[(4C_2x - 4C_1y)X_1 + (4C_0y - 4C_2)X_2 + (-4C_0x + 4C_1)X_3]X_0 \quad +[(4C_1 + 4C_0x)X_2 + (4C_0y + 4C_2)X_3]X_1 - 4C_0X_3^2 - 4C_0X_2^2 = 0.$$

2.3 Mechanism Synthesis

We then satisfy the Poses in the Constraint equation to generate Dyad parameters. First we transform poses such y=that one of the poses align with the origin. Thus we are left with only 4 poses. We then eliminate the R in constraints.

$$(X_0 : X_1 : X_2 : X_3) = (1 : 0 : 0 : 0)$$

$$R^2 - C_1^2 - C_2^2 - C_0(x^2 + y^2) + 2C_1 x + 2C_2 y = 0$$

$$(-X_0X_3x + X_0X_2y + X_1X_2x + X_3X_1y - X_2^2 - X_3^2)C_0 - X_0X_2C_2 + X_0X_3C_1 + X_0X_1xC_2 - X_1^2xC_1 + X_1X_2C_1 - X_0X_1yC_1 - X_1^2yC_2 + X_1X_3C_2 = 0$$

Case 1:- At least one point lie on a line i.e. Co=0

$$-X_0X_2C_2 + X_0X_3C_1 + X_0X_1xC_2 - X_1^2xC_1 + X_1X_2C_1 - X_0X_1yC_1 - X_1^2yC_2 + X_1X_3C_2 = 0$$

$$[(-C_2x + C_1y)X_{1i} - C_1X_{3i} + C_2X_{2i}]X_{0i} + (C_2y + C_1x)X_{1i}^2 + (-C_2X_{3i} - C_1X_{2i})X_{1i} = 0 \qquad i = 1, \ldots, 4.$$

We get the following conditions. E1 and E2 are applicable when slider is parallel to x axis while E2 is applicable bor any orientation of slider

$$E_1: \quad \left(-\frac{X_{13}(-X_{11}^2X_{02}X_{22} + X_{01}X_{21}X_{12}^2 - X_{11}X_{31}X_{12}^2 + X_{11}^2X_{12}X_{32})}{X_{11}X_{12}(X_{01}X_{12} - X_{11}X_{02})} + X_{23}\right)X_{03} - \frac{(X_{11}X_{31}X_{02}X_{12} - X_{01}X_{11}X_{12}X_{32} - X_{01}X_{21}X_{02}X_{12} + X_{01}X_{11}X_{02}X_{22})X_{13}^2}{X_{12}X_{11}(X_{01}X_{12} - X_{11}X_{02})} - X_{13}X_{33} = 0$$

$$E_2: \quad \left(-\frac{X_{14}(-X_{11}^2X_{02}X_{22} + X_{01}X_{21}X_{12}^2 - X_{11}X_{31}X_{12}^2 + X_{11}^2X_{12}X_{32})}{X_{11}X_{12}(X_{01}X_{12} - X_{11}X_{02})} + X_{24}\right)X_{04} - \frac{(X_{11}X_{31}X_{02}X_{12} - X_{01}X_{11}X_{12}X_{32} - X_{01}X_{21}X_{02}X_{12} + X_{01}X_{11}X_{02}X_{22})X_{14}^2}{X_{12}X_{11}(X_{01}X_{12} - X_{11}X_{02})} - X_{34}X_{14} = 0$$

$$
\begin{aligned}
E_3: \quad (&X_{14}X_{04}X_{01}X_{02}X_{31}X_{13}^2X_{22} - X_{34}X_{14}X_{03}X_{13}X_{12}^2X_{21}X_{11} - X_{34}X_{04}X_{13}^2X_{11}X_{01}X_{02}X_{22} - X_{34}X_{04}X_{13}X_{11}X_{12}^2X_{33}X_{01} \\
&+ X_{34}X_{04}X_{03}X_{23}X_{11}X_{12}^2X_{01} + X_{34}X_{04}X_{13}^2X_{11}X_{01}X_{12}X_{32} + X_{34}X_{04}X_{13}^2X_{02}X_{12}X_{01}X_{21} - X_{34}X_{04}X_{03}X_{13}X_{12}^2X_{01}X_{21} \\
&+ X_{34}X_{04}X_{13}X_{11}^2X_{12}X_{33}X_{02} - X_{34}X_{04}X_{03}X_{23}X_{11}^2X_{12}X_{02} + X_{34}X_{04}X_{03}X_{13}X_{11}^2X_{02}X_{22} - X_{34}X_{04}X_{03}X_{13}X_{11}^2X_{12}X_{32} \\
&- X_{34}X_{04}X_{13}^2X_{02}X_{12}X_{11}X_{31} + X_{34}X_{04}X_{03}X_{13}X_{12}^2X_{11}X_{31} - X_{14}^2X_{11}X_{02}X_{32}X_{31}X_{03}X_{13} - X_{14}^2X_{11}X_{12}X_{03}X_{31}X_{13}X_{22} \\
&+ X_{14}^2X_{11}X_{12}X_{32}X_{03}X_{21}X_{13} - X_{14}X_{04}X_{01}X_{02}X_{32}X_{21}X_{13}^2 + X_{14}X_{04}X_{01}X_{12}^2X_{13}X_{23}X_{21} + X_{14}X_{04}X_{01}X_{12}^2X_{13}X_{31}X_{33} \\
&- X_{14}X_{04}X_{01}X_{12}^2X_{31}X_{03}X_{23} + X_{14}X_{04}X_{01}X_{12}^2X_{03}X_{33}X_{21} - X_{14}X_{04}X_{01}X_{12}X_{21}X_{13}^2X_{22} - X_{14}X_{04}X_{01}X_{12}X_{31}X_{32}X_{13}^2 \\
&- X_{14}X_{04}X_{11}^2X_{02}X_{22}X_{23}X_{13} - X_{14}X_{04}X_{11}^2X_{02}X_{22}X_{03}X_{33} - X_{14}X_{04}X_{11}^2X_{02}X_{32}X_{33}X_{13} + X_{14}X_{04}X_{11}^2X_{02}X_{32}X_{03}X_{23} \\
&- X_{14}X_{04}X_{11}^2X_{12}X_{22}X_{33}X_{13} + X_{14}X_{04}X_{11}^2X_{12}X_{22}X_{03}X_{23} + X_{14}X_{04}X_{11}^2X_{12}X_{32}X_{23}X_{13} + X_{14}X_{04}X_{11}^2X_{12}X_{32}X_{03}X_{33} \\
&+ X_{14}X_{04}X_{11}X_{02}X_{21}X_{13}^2X_{22} + X_{14}X_{04}X_{11}X_{02}X_{31}X_{32}X_{13}^2 + X_{14}X_{04}X_{11}X_{12}^2X_{13}X_{33}X_{21} - X_{14}X_{04}X_{11}X_{12}^2X_{13}X_{31}X_{23} \\
&- X_{14}X_{04}X_{11}X_{12}^2X_{21}X_{23}X_{03} - X_{14}X_{04}X_{11}X_{12}^2X_{31}X_{03}X_{33} + X_{14}X_{04}X_{11}X_{12}X_{31}X_{13}^2X_{22} - X_{14}X_{04}X_{11}X_{12}X_{32}X_{21}X_{13}^2 \\
&- X_{24}X_{14}X_{13}^2X_{11}X_{01}X_{02}X_{22} - X_{24}X_{14}X_{13}X_{11}X_{12}^2X_{33}X_{01} + X_{24}X_{14}X_{03}X_{23}X_{11}X_{12}^2X_{01} + X_{24}X_{14}X_{13}^2X_{11}X_{01}X_{12}X_{32} \\
&+ X_{24}X_{14}X_{13}^2X_{02}X_{12}X_{01}X_{21} - X_{24}X_{14}X_{03}X_{13}X_{12}^2X_{01}X_{21} + X_{24}X_{14}X_{13}X_{11}^2X_{12}X_{33}X_{02} - X_{24}X_{14}X_{03}X_{23}X_{11}^2X_{12}X_{02} \\
&+ X_{24}X_{14}X_{03}X_{13}X_{11}^2X_{02}X_{22} - X_{24}X_{14}X_{03}X_{13}X_{11}^2X_{12}X_{32} - X_{24}X_{14}X_{13}^2X_{02}X_{12}X_{11}X_{31} + X_{24}X_{14}X_{03}X_{13}X_{12}^2X_{11}X_{31} \\
&- X_{34}X_{14}X_{13}^2X_{11}X_{01}X_{02}X_{32} + X_{34}X_{14}X_{13}X_{11}X_{12}^2X_{23}X_{01} + X_{34}X_{14}X_{03}X_{33}X_{11}X_{12}^2X_{01} - X_{34}X_{14}X_{13}^2X_{11}X_{01}X_{12}X_{22} \\
&+ X_{34}X_{14}X_{13}^2X_{02}X_{12}X_{31}X_{01} - X_{34}X_{14}X_{03}X_{13}X_{12}^2X_{31}X_{01} - X_{34}X_{14}X_{13}X_{11}^2X_{12}X_{23}X_{02} - X_{34}X_{14}X_{03}X_{33}X_{11}^2X_{12}X_{02} \\
&+ X_{34}X_{14}X_{03}X_{13}X_{11}^2X_{02}X_{32} + X_{34}X_{14}X_{03}X_{13}X_{11}^2X_{12}X_{22} + X_{34}X_{14}X_{13}^2X_{02}X_{12}X_{21}X_{11} + X_{24}X_{04}X_{03}X_{13}X_{12}^2X_{21}X_{11} \\
&+ X_{24}X_{04}X_{13}^2X_{11}X_{01}X_{02}X_{32} + X_{24}X_{04}X_{13}^2X_{11}X_{01}X_{12}X_{22} - X_{24}X_{04}X_{13}^2X_{02}X_{12}X_{31}X_{01} - X_{24}X_{04}X_{13}^2X_{02}X_{12}X_{21}X_{11} \\
&+ X_{24}X_{04}X_{03}X_{33}X_{11}^2X_{12}X_{02} - X_{24}X_{04}X_{03}X_{13}X_{11}^2X_{02}X_{32} + X_{24}X_{04}X_{03}X_{13}X_{12}^2X_{31}X_{01} - X_{24}X_{04}X_{03}X_{13}X_{11}^2X_{12}X_{22} \\
&- X_{24}X_{04}X_{03}X_{33}X_{11}X_{12}^2X_{01} + X_{24}X_{04}X_{13}X_{11}^2X_{12}X_{23}X_{02} - X_{24}X_{04}X_{13}X_{11}X_{12}^2X_{23}X_{01} + X_{14}^2X_{01}X_{11}X_{02}X_{22}X_{23}X_{13} \\
&+ X_{14}^2X_{01}X_{11}X_{02}X_{22}X_{03}X_{33} + X_{14}^2X_{01}X_{11}X_{02}X_{32}X_{33}X_{13} - X_{14}^2X_{01}X_{11}X_{02}X_{32}X_{03}X_{23} + X_{14}^2X_{01}X_{11}X_{12}X_{22}X_{33}X_{13} \\
&- X_{14}^2X_{01}X_{11}X_{12}X_{22}X_{03}X_{23} - X_{14}^2X_{01}X_{11}X_{12}X_{32}X_{23}X_{13} - X_{14}^2X_{01}X_{11}X_{12}X_{32}X_{03}X_{33} - X_{14}^2X_{01}X_{02}X_{12}X_{13}X_{31}X_{33} \\
&- X_{14}^2X_{01}X_{02}X_{12}X_{13}X_{21}X_{23} - X_{14}^2X_{01}X_{02}X_{12}X_{21}X_{03}X_{33} + X_{14}^2X_{01}X_{02}X_{12}X_{31}X_{03}X_{23} - X_{14}^2X_{01}X_{02}X_{03}X_{31}X_{13}X_{22} \\
&+ X_{14}^2X_{01}X_{02}X_{32}X_{03}X_{21}X_{13} + X_{14}^2X_{01}X_{12}X_{03}X_{21}X_{13}X_{22} + X_{14}^2X_{01}X_{12}X_{32}X_{03}X_{31}X_{13} - X_{14}^2X_{11}X_{02}X_{12}X_{13}X_{33}X_{21} \\
&+ X_{14}^2X_{11}X_{02}X_{12}X_{13}X_{31}X_{23} + X_{14}^2X_{11}X_{02}X_{12}X_{21}X_{23}X_{03} + X_{14}^2X_{11}X_{02}X_{12}X_{31}X_{03}X_{33} - X_{14}^2X_{11}X_{02}X_{21}X_{03}X_{13}X_{22}) = 0
\end{aligned}
$$

Case 1:-Point lie on a circle i.e. Co=1

The equations can be simplified by substituing b1,b2,b3,b4. Then the 4 equations we get have to be solved to get the dyads.

$$
\begin{aligned}
((-C_1y + C_2x)X_1 + (y - C_2)X_2 + (C_1 - x)X_3)X_0 + (-C_2y - C_1x)X_1^2 \\
+((x + C_1)X_2 + (y + C_2)X_3)X_1 - X_3^2 - X_2^2 = 0.
\end{aligned}
$$

$$
\begin{aligned}
-x - C_1 &= 2b_1 & \qquad y - C_2 &= 2b_3 \\
-y - C_2 &= 2b_2 & \qquad -x + C_1 &= 2b_4.
\end{aligned}
$$

$$
\begin{aligned}
[(2b_1b_3 + 2b_2b_4)X_{1i} + 2b_3X_{2i} + 2b_4X_{3i}]X_{0i} + (-b_1^2 - b_2^2 + b_3^2 + b_4^2)X_{1i}^2 \\
+(-2b_1X_{2i} - 2b_2X_{3i})X_{1i} - X_{2i}^2 - X_{3i}^2 = 0 \\
i = 1, \ldots, 4.
\end{aligned}
$$

Chapter 3

Algorithm

1. Initially five poses of a moving system E are given.

2. Apply a coordinate transformation to all given poses such that one of the poses coincides with the fixed coordinate frame.

3. Now only four arbitrary poses of a moving system remain, given by their coordinates (X0i : X1i : X2i : X3i) for i = 1, ... ,4 in the kinematic image space.

4. To determine if all poses lie in the same assembly mode.

5. Substitute the coordinates of the poses in the Equations E1, E2 and E3 to determine if some of the four Burmester points we are searching for are at infinity.

   (a) E1, E2 and E3 are satisfied: at least two Burmester points are at infinity and one of them is the point at infinity of the Y-axis of the fixed frame.

   (b) Only E1 and E2 are satisfied: only one Burmester point is a point at infinity. This point is the point at infinity of the X-axes of the fixed frame.

   (c) Only E3 is satisfied: only one Burmester point is a point at infinity. This point is not the point at infinity of the X-axes of the fixed frame.

   (d) None of the conditions E1, E2 and E3 is satisfied: none of the Burmester points is at infinity.

6. Substitute the input data into the general case equation to obtain the finite solutions.

Chapter 4

Implementation

4.1 Implementation in Mathemetica

The code uses equations obtained from Mathematica by:

1. In case of Slider Crank and Slider parallel to X-axis of Fixed co-ordinate system: Solving 2 equations of a system of 4 linear equations in x, y and back substituting the solutions into the other two equations yields two compatibility conditions E1 and E2.

2. In case of Slider Crank and Slider not parallel to X-axis of Fixed co-ordinate system: Solving 2 out of a system of 4 linear equations in three variables x, y and C2 linearly for x and y and substituting the solutions into one of the remaining equations an equation of degree 3 in C2 is obtained, which can be factored. Two solutions of this equation are complex and do not correspond to real mechanisms. The other factor of the equation can be solved linearly for C2. Substituting the solutions for x, y and C2 into the remaining fourth equation yields a compatibility condition E3.

3. In case of General 4 Bar: Applying linear coordinate transformation to circle constraint equation yields a system of 4 equations which are used to derive a closed form solution. The quadratic terms of the unknowns in the constraint equations are eliminated by simple manipulations this yields 3 bilinear equations in the unknowns b1, b2, b3 , b4. Repeating the same process we get rid of the bilinear terms and 2 equations linear in the unknowns remain. Solving these equations for two of the unknowns, e.g. b1, b2, gives these two as functions of b3 and b4. Taking one of the bilinear equations and one out of the system dependent just on b3 and b4 and calculating the resultant yields an equation of degree four in b4. This equation can be solved in closed form. It has 0, 2 or 4 real roots for b4. Substituting one of this values in the two equations used for the resultant yields two quadratic equations in b3. By eliminating (b3) we get one linear equation in b3. We can use the same procedure to obtain one value of b3 for each real root of b4. As b1 and b2 are linear functions of b3 and b4 they can be calculated easily. With the inverse coordinate transformation we obtain all the unknowns.

4.2 Implementation in c++

The code has been implemented using 4MDS software created under the supervision of Prof Purwar. We use the softwares GUI to interactively input and display the output. The research papers algorithm has been implemented within a function which interacts with the software.

1. We can input poses using GUI or text file. Text files were used as input to verify the result presented in paper. If the text file is empty, the program automatically takes points from GUI. As the points are inputted, our code sets motion class variables accordingly. Also, the coupler point is set to the first position.

2. Once the number of poses inputted reaches 5, the program displays a dialog prompting the user to run the Mathematica code which inputs poses and generates Dyads and compatibility variables.

3. After running mathematica code, the dialog is closed and our code automatically calculates if the dyad type is RR or PR. It then goes on to initialize Dyads lists and planer4barmechanism classes within 4MDS to display the dyads on the GUI as output.

Chapter 5

Results

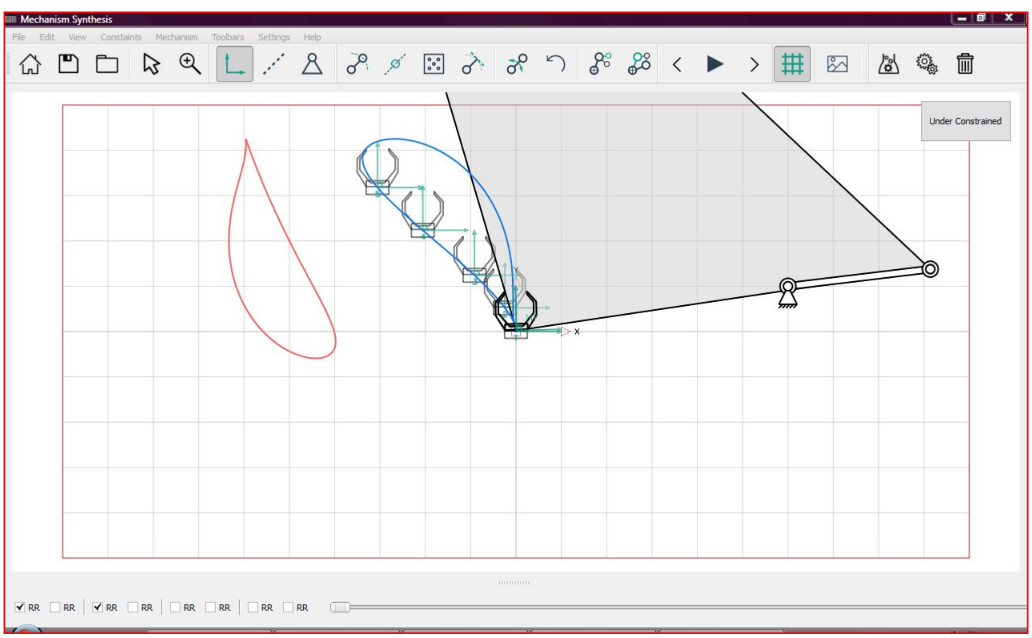
The Examples implemented in the research paper are verified using our algorithm. Also Poses have been taken tfrom user to generate dyads accordingly.

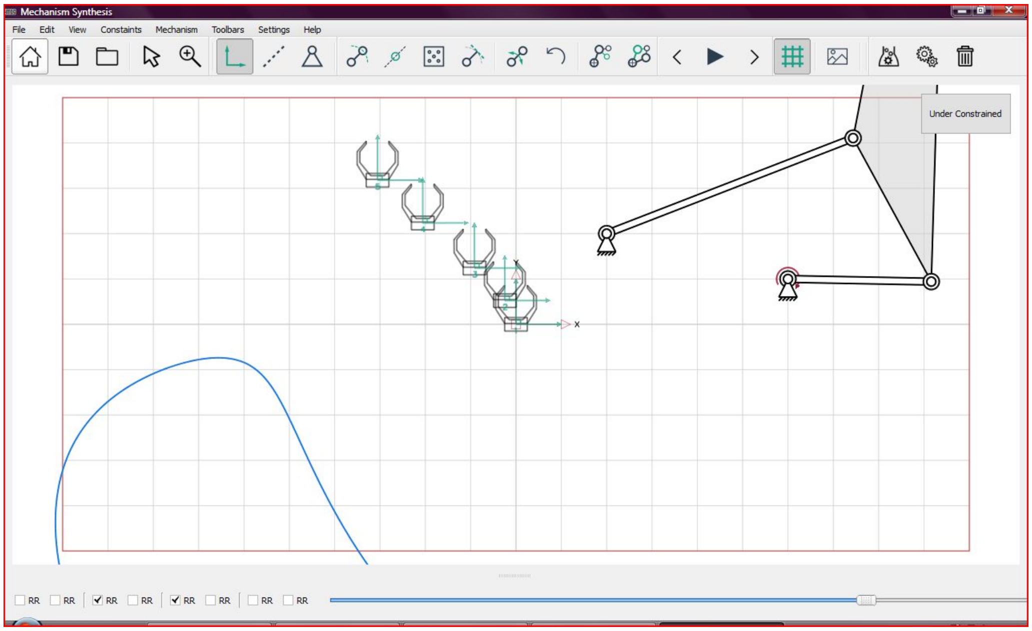
5.1 RR-RR Dyad

The following Dyads are generated by Mathematica code.

-4.402224956706082	5.999674806112333	2.0002428765454447	-34.639102419585555
16.135828166521947	1.0001657009914733	2.000023543729121	-29.946087279571522
-3.697500592552355	9.160247457887428	7.382138244900643	18.091189365523032
13.877101174971017	1.107249598692889	4.24327478403945	17.843912130261607

The following Plots show two of the mechanisms formed by the Dyads



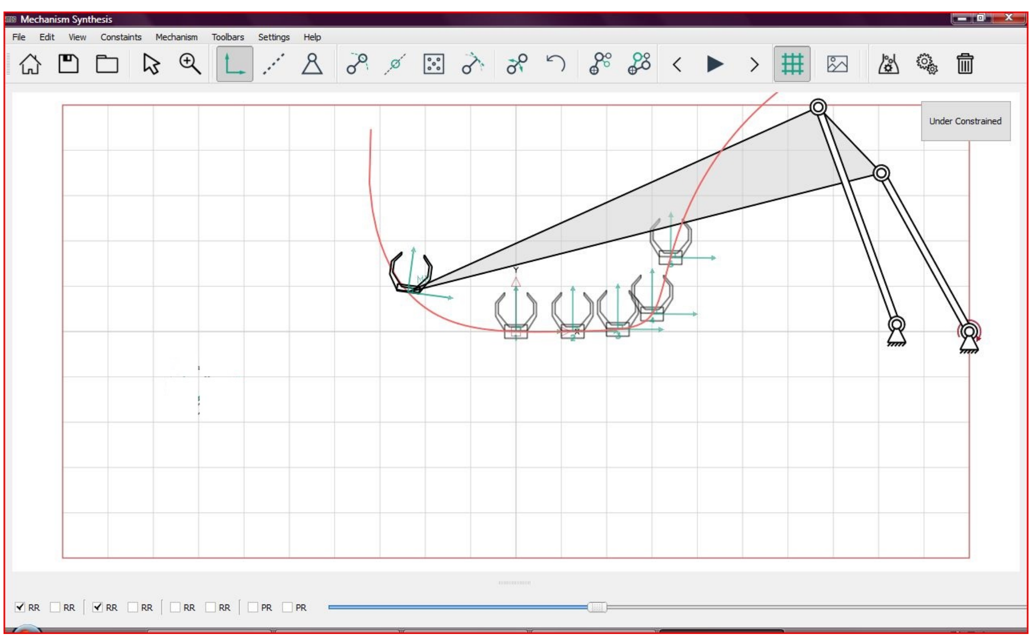


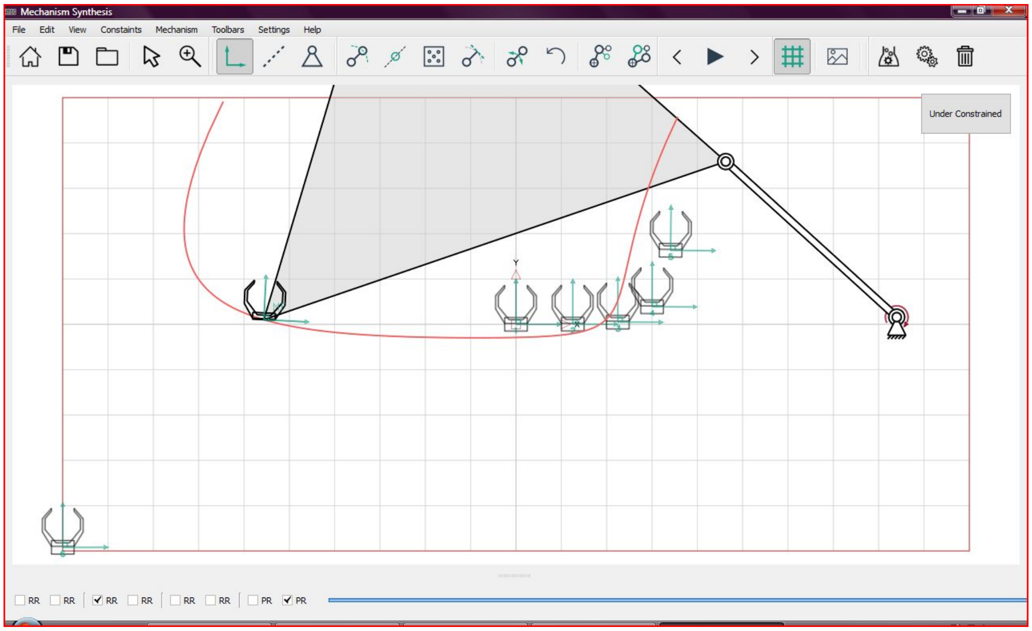
5.2 RR-PR Dyad Parallel to X axis

The following Dyads are generated by Mathematica code.

10.000004750246685	8.398922846225174	0.6953172742166408	-0.6942584757679522
0.000029228247160340004	0.15831871725372704	-57.11667181435004	-797301.9036810105
10.00002239568163	8.42560075612906	0.850308625588771	0.0000932637449576057
4.0000856386787325	5.258013043622042	11.310355883136538	10.000038809957914

The following Plots show two of the mechanisms formed by the Dyads



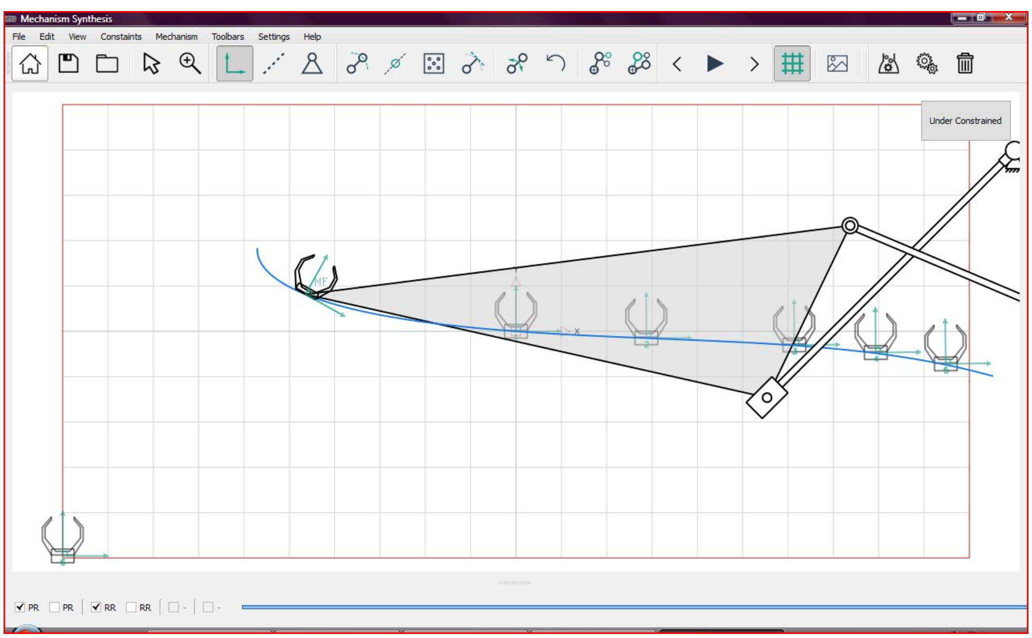


5.3 RR-PR Dyad not Parallel to X axis

The following Dyads are generated by Mathematica code.

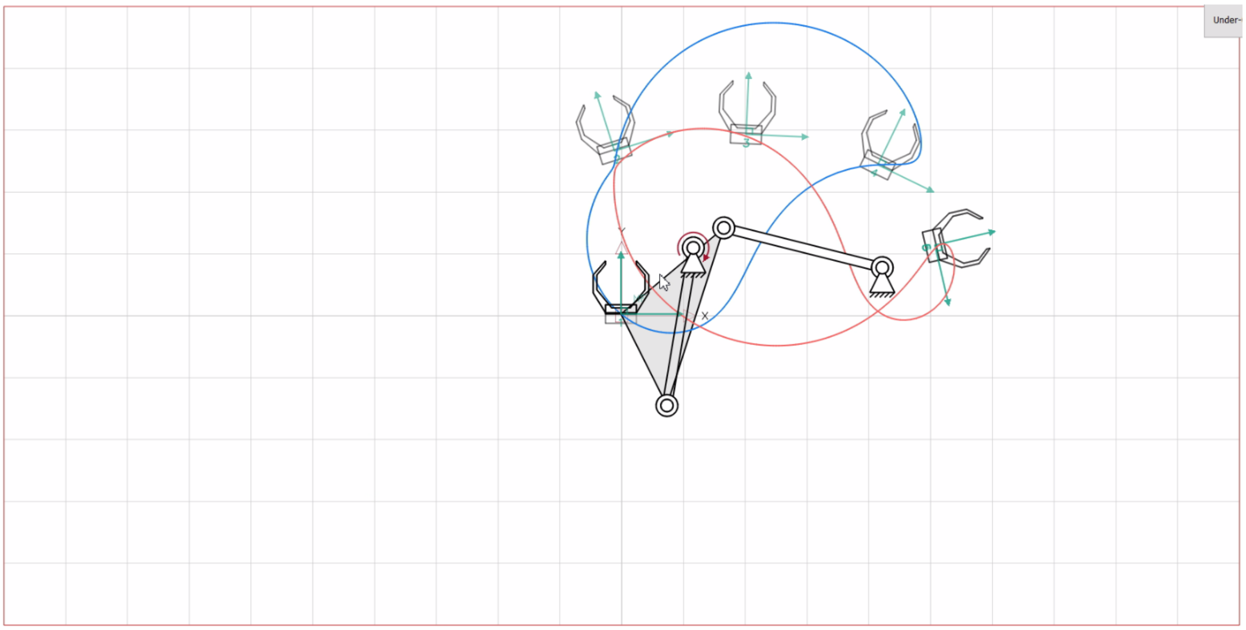
-17397.66267983932	53.76053653632995
17438.114298231503	-17.294607747715805
9.998820252481892	9.743927333549518
2.9866447950917063	7.19388780768917

The following Plot show the mechanisms formed by the Dyads



5.4 Arbitrary poses input from 4MDS

The user can enter points in 4MDS and using Mathematica, the dyads will be displayed.



Chapter 6

Conclusion

1. 5 Pose Burmister Problem for Dyad generation has been solved according to the algorithms propesed by the research paper.

2. The results presented in the paper have been cross verified with the program implemented.

3. Points inputted by user have been used to generate dyads using the 4MDS software.

6.1 Future Scope

1. Restore the compromised functionality of the program like Edit Coupler and Zoom out/in.

2. Debug to prevent errors.

3. Scale it to all 5 poses.

Chapter 7

References

1. K. Brunnthaler, M. Pfurner, and M. Husty. Synthesis of planar four-bar mechanisms
2. Professor Anurag Purwar, 4MDS software